Planck Units in 11 Dimensions

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Abstract

Planck units are derived from five physical constants (Gravitational constant, Planck constant, Speed of light, Coulomb constant, Boltzmann constant) using dimensional analysis. Despite their popularity among physicists the units have no practical use. They are notorious for wild swing of orders of magnitude in respect to observed physical phenomena.

For example, Planck length **$1.616199\ldots\cdot10^{-35}$ m** is 20 orders smaller than proton charge radius while Planck mass is **$2.17651\ldots\cdot10^{-8}$ kg** is 19 orders bigger than proton mass.

Planck Temperature **$1.417\ldots\cdot10^{32}$ K** is so high that Big Bang temperature **10^{13} K** is very cold event.

This article describes how Planck units can be normalized by deriving them in consideration of 11 dimensions in our space-time continuum.

Three spatial dimensions we see and the time we feel are not enough to elucidate sizes, masses and other properties of elementary particles. But in 11 dimensions all Planck units suddenly fit the proton scale.

Newton's law of gravitation in 11 dimensions

Experimentally confirmed Newton's law of gravitation gives no doubt in its validity on human scale

$$\mathbf{F} = -\boldsymbol{G}\cdot\frac{\boldsymbol{M}\cdot\boldsymbol{m}}{\boldsymbol{r}^2} \qquad [1]$$

where

- **F** - force of gravitation between to masses ***M*** and ***m***;
- ***G*** - gravitational constant **$6.67384..\cdot10^{-11}$ $m^3\cdot kg^{-1}\cdot s^{-2}$**;
- ***r*** – distance between masses.

According to formula [1] Gravitational force is inverse proportional to the square of a distance between two masses. But is this law applicable for all distances? What if for distances smaller than size of an atom the gravitational force becomes inverse proportional to the 9-th power of a distance?

$$\mathbf{F} = -\,\mathbb{\Gamma} \cdot \frac{\boldsymbol{M \cdot m}}{\boldsymbol{r^9}} \qquad [2]$$

- where $\mathbb{\Gamma}$ - gravitational constant for distances smaller than size of an atom (a size of an atom can be characterized by Bohr radius $a_0 = 5.2917721092... \cdot 10^{-11}$ m).

9-th power of a distance in equation [2] corresponds to 11 dimensions similar to 2-d power of a distance in equation [1] for 4 dimensions (3 spatial dimensions and the time).

In other words, as a distance gets smaller, curved seven extra dimensions become flat on this scale and contribute more to the force of gravitation. All 11 dimensions work and gravitation becomes stronger. Is it “strong force” itself?

To make estimation for gravitational constant $\mathbb{\Gamma}$ for subatomic distances let assume that forces in equation [1] and equation [2] are equal to each other at a distance of Bohr radius a_0. Thus,

$$\mathbb{\Gamma} = \boldsymbol{G \cdot a_0^7} \qquad [3]$$

It has value of $7.7550... \cdot 10^{-83}$ $\mathbf{m^{10} \cdot kg^{-1} \cdot s^{-2}}$. Interpolation plot Fig.1 shows that gravitational constant for subatomic distances should be bigger than value $\mathbb{\Gamma}$ calculated from equation [3]. If transition (shown as dashed line) from $\sim 1/r^2$ to $\sim 1/r^9$ law of gravitation happens in the range half of Bohr radius, than magnitude of gravitational constant $\mathbb{\Gamma}^*$ will be closer to $2.0564... \cdot 10^{-82}$ $\mathbf{m^{10} \cdot kg^{-1} \cdot s^{-2}}$.

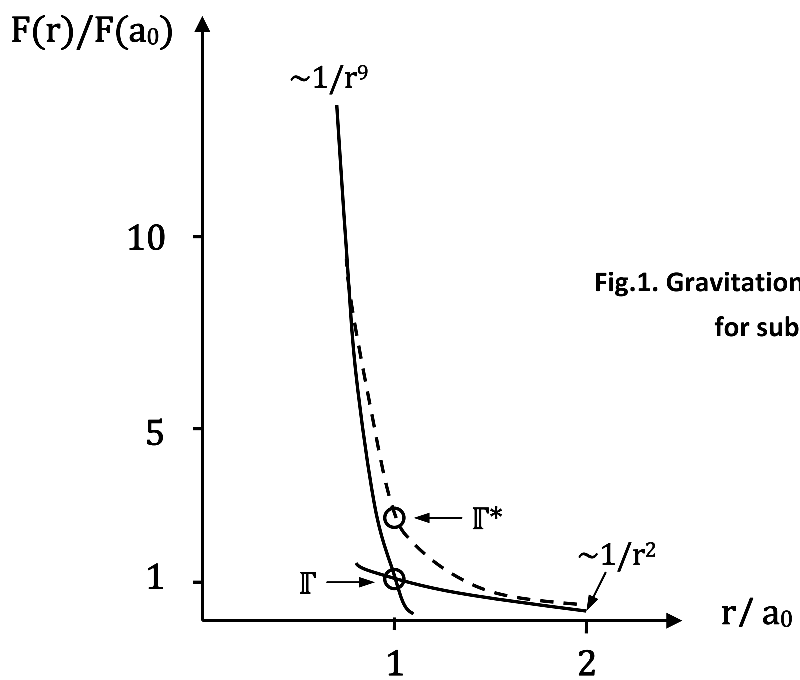

Fig.1. Gravitational force F vs. distance r for subatomic range.

Dimensional analysis for normalized Planck units in 11 dimensions

New normalized Planck units for 11 dimensions can be obtained from next physical constants using dimensional (referring to measuring units) analysis

c - speed of light $2.99792458 \cdot 10^{8}$ **m ·s**$^{-1}$

$\mathbb{\Gamma}$ **-** subatomic gravitational constant from equation [3] $7.7550... \cdot 10^{-83}$ **m**^{10}**·kg**$^{-1}$**·s**$^{-2}$ (instead of Newton gravitational constant ***G***);

$\hbar$ - reduced Planck constant $1.054571726... \cdot 10^{-34}$ **J ·s**

ε_0 - permittivity of free space $8.854187817620... \cdot 10^{-12}$ **C ·N**$^{-1}$ **·m**$^{-2}$

k_B - Boltzmann constant $1.3806488... \cdot 10^{-23}$ **J ·K**$^{-1}$

Results of dimensional analysis are next.

Normalized Planck length

$$\boldsymbol{l_N = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}}{c^3}} = 1.887121 \ldots \cdot 10^{-16}\, m} \qquad [4]$$

It is an order of reduced Compton wavelength for proton.

Normalized Planck mass

$$\boldsymbol{m_N = \sqrt[9]{\frac{\hbar^8}{\mathbb{\Gamma} \cdot c^6}} = 1.864042 \ldots \cdot 10^{-27}\, kg} \qquad [5]$$

It is an order of mass for proton.

Normalized Planck time

$$t_N = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}}{c^{12}}} = 6.294758 \ldots \cdot 10^{-25}\ s \qquad [6]$$

Time required for light to travel a distance about Compton wavelength for proton.

Normalized Planck charge

$$q_N = \sqrt[2]{4\pi\varepsilon_0 \cdot \hbar \cdot c} = 1.875\,545\,956 \ldots \cdot 10^{-18}\ C \qquad [7]$$

It is the same as original Planck charge (approximately 11.7 charges of proton or electron).

Normalized Planck temperature

$$T_N = \frac{1}{k_B} \cdot \sqrt[9]{\frac{\hbar^8 \cdot c^{12}}{\mathbb{\Gamma}}} = 1.213428 \ldots \cdot 10^{13}\ K \qquad [8]$$

It is an order of Big Bang temperature during first second.

Other normalized Planck units in 11 dimensions

Normalized Planck area

$$l_N^2 = \sqrt[9]{\frac{\hbar^2 \cdot \mathbb{\Gamma}^2}{c^6}} = 3.5612257 \ldots \cdot 10^{-32}\ m^2 \qquad [9]$$

It is an order of surface of proton divided by (**2 ·π**)^2.

Normalized Planck volume

$$l_N^3 = \sqrt[9]{\frac{\hbar^3 \cdot \mathbb{\Gamma}^3}{c^9}} = 6.72046379 \ldots \cdot 10^{-48}\ m^3 \qquad [10]$$

It is an order of volume of proton divided by (**2 ·π**)^3.

Normalized Planck momentum

$$m_N \cdot c = \sqrt[9]{\frac{\hbar^8 \cdot c^3}{\mathbb{\Gamma}}} = 5.5882571 \ldots \cdot 10^{-19}\ kg \cdot m \cdot s^{-1} \qquad [11]$$

It is an order of momentum for proton moving at speed close to that of light.

Normalized Planck energy

$$E_N = m_N \cdot c^2 = \sqrt[9]{\frac{\hbar^8 \cdot c^{12}}{\mathbb{\Gamma}}} = 1.6753173 \ldots \cdot 10^{-10}\ J \qquad [12]$$

It is an order of energy for proton at rest.

Normalized Planck force

$$F_N = \frac{E_N}{l_N} = \sqrt[9]{\frac{\hbar^7 \cdot c^{15}}{\mathbb{\Gamma}^2}} = 8.877636 \ldots \cdot 10^5\ N \qquad [13]$$

It is an order of energy for proton at rest divided by normalized Planck length.

Normalized Planck power

$$P_N = \frac{E_N}{t_N} = \sqrt[9]{\frac{\hbar^7 \cdot c^{24}}{\mathbb{\Gamma}^2}} = 2.661448 \ldots \cdot 10^{14}\ W \qquad [14]$$

It is an order of energy for proton at rest divided by normalized Planck time.

Normalized Planck density

$$\boldsymbol{\rho_N = \frac{m_N}{l_N^3} = \sqrt[9]{\frac{\hbar^7 \cdot c^3}{\mathbb{\Gamma}^4}} = 2.77368 \ldots \cdot 10^{20}\ kg \cdot m^{-3}} \qquad [\mathbf{15}]$$

It is an order of neutron star density multiplied by $(\mathbf{2 \cdot \pi})^3$.

Normalized Planck angular frequency

$$\boldsymbol{\omega_N = \frac{1}{t_N} = \sqrt[9]{\frac{c^{12}}{\hbar \cdot \mathbb{\Gamma}}} = 1.588623 \ldots \cdot 10^{24}\ s^{-1}} \qquad [\mathbf{16}]$$

It is an order of frequency for proton.

Normalized Planck pressure

$$\boldsymbol{p_N = \frac{F_N}{l_N^2} = \sqrt[9]{\frac{\hbar^5 \cdot c^{21}}{\mathbb{\Gamma}^4}} = 2.492859 \ldots \cdot 10^{37}\ Pa} \qquad [\mathbf{17}]$$

It is an order of normalized Planck force applied to a square with side equal normalized Planck length.

Normalized Planck current

$$\boldsymbol{I_N = \frac{q_N}{t_N} = \sqrt[2]{4\pi\varepsilon_0} \cdot \sqrt[9]{\frac{\hbar^{\frac{7}{2}} \cdot c^{\frac{33}{2}}}{\mathbb{\Gamma}}} = 2.979536 \ldots \cdot 10^{6}\ A} \qquad [\mathbf{18}]$$

It is Planck charge divided by normalized Planck time.

Normalized Planck voltage

$$\boldsymbol{V_N = \frac{E_N}{q_N} = \frac{1}{\sqrt[2]{4\pi\varepsilon_0}} \cdot \sqrt[9]{\frac{\hbar^{\frac{7}{2}} \cdot c^{\frac{15}{2}}}{\mathbb{\Gamma}}} = 8.932424 \ldots \cdot 10^7\, V} \qquad [19]$$

It is normalized Planck energy divided by Planck charge.

Normalized Planck impedance

$$\boldsymbol{Z_N = \frac{V_N}{I_N} = \frac{1}{4\pi\varepsilon_0 c} = 29.9792458 \ldots\, \Omega} \qquad [20]$$

It is the same as original Planck impedance.

Is proton a black hole?

If gravity obeys the $\sim 1/r^9$ law (equation [2]) and distance r becomes smaller and smaller the gravity eventually surpasses the repelling force of electrical charges of the same polarity. For two particles with normalized Planck masses m_N (equation [5]) and Planck charges q_N (equation [7]) the equilibrium between gravitational and electrical forces happens at the distance of normalized Planck length l_N (equation [4]) . For two proton charges the equilibrium distance equals to the normalized Planck length l_N (equation [4]) multiplied by 7-th root of reciprocal of the Fine structure constant α. This factor is approximately equal 2.02…

$$\boldsymbol{l_{Gr=El} = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}^*}{c^3}} \cdot \frac{1}{\sqrt[7]{\alpha}} = l_N \cdot 2.02 \ldots} \qquad [21]$$

- where Fine structure constant $\alpha = 1/137.0359991\ldots$.

Let calculate energy required for particle of mass m to escape another particle of mass M using gravitational $\sim 1/\mathrm{r}^9$ law from equation [2]

$$\int_{\mathrm{R}_s}^{\infty} \mathbf{F} \cdot \mathbf{dr} = -\int_{R_s}^{\mathrm{F}} \mathbb{\Gamma} \cdot \frac{M \cdot m}{r^9} \cdot dr = \mathbb{\Gamma} \cdot \frac{M \cdot m}{8 \cdot R_s^8} \qquad [22]$$

- where $\mathrm{R_s}$ is initial distance between particles .

Equalizing energy [22] with energy $\mathrm{m \cdot c^2}$ the distance $\mathrm{R_s}$ can be expressed as follows.

$$R_s = \sqrt[8]{\mathbb{\Gamma} \cdot \frac{M}{8 \cdot c^2}} = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}^*}{c^3}} \cdot \frac{1}{\sqrt[8]{8}} = l_N \cdot 0.771 \ldots \qquad [23]$$

This is Schwarzschild radius for normalized Planck mass. Thus, proton can be a self sustaining black hole due to self gravity.

Conclusion

1. The major result of research is deriving of mass value (equation [5]) close to that of proton from physical constants not containing information about proton.
   Using interpolated (Fig.1) value of $2.0564\ldots\cdot 10^{-82}$ $\mathbf{m^{10} \cdot kg^{-1} \cdot s^{-2}}$ for gravitational constant $\mathbb{\Gamma}^*$ instead of value $\mathbb{\Gamma} = 7.7550\ldots\cdot 10^{-83}$ $\mathbf{m^{10} \cdot kg^{-1} \cdot s^{-2}}$ from equation [3] the normalized Planck mass is equalized with the mass of proton and normalized Planck length is equalized with Compton wavelength for proton.

2. If gravitational law for subatomic distances can be described by equation [2] than “strong” force is just gravitation, because for distances shorter than normalized Planck length the gravitation exceeds electrostatic force.

3. Proton can be a black hole with radius about normalized Planck length (equation [23]).

References

1. Duff, Michael J., M-Theory (the Theory Formerly Known as Strings), International Journal of Modern Physics A, 11 (1996) 5623–5642, online at Cornell University's arXiv ePrint server