Physical Constants in a Mirror of Extra Dimensions

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Abstract

Philosophical question "Do extra dimensions exist in our space-time continuum?" has two short answers: trivial "No" and debatable "Yes".

If extra dimensions are real why we cannot see them? It is possible that extent of human senses is not sufficient to observe all directions in space. We can only feel 3 spatial dimensions and the Time. This is our natural boundary. But scientists gradually construct increasingly sophisticated devices to quantify different physical phenomena thus expanding the borders of knowledge.

The progress in physics is especially obvious in the high precision measurements of the fundamental physical constants. With an exception of Gravitational constant (which is difficult to determine precisely due to weakness of gravity) the majority of the fundamental physical constants are known now with uncertainty less than one part per billion. Unprecedented accuracy of data for constant's numerical values creates an opportunity for mathematical explanation of nature's plan on gravity, particle masses and electromagnetic interactions.

The article describes the derivations of the values of the fundamental physical constants through equations based solely on the number of spatial dimensions and their differentiation. It is shown in the article that the values of the Fine Structure constant, Proton-to-electron mass ratio and Gravitational constant can be calculated by taking 7 additional spatial dimensions into account.

The above statement can be mirrored as follows: the values of the fundamental physical constants point to the existence of extra 7 dimensions making total number of spatial dimensions 10. Presented transformation for expressions and values of the Planck units may also serve as an additional confirmation for the hypothesis that our space really extents to 10 dimensions from 3 dimensions we observe.

Part 1. Where number 137 comes from in the integer portion of the Fine Structure constant reciprocal and how to get constant's precise value?

Physicists should admit: the Fine Structure constant continues to attract attention of scientific community mostly because of constant's unexplainable numerical value. Its unbreakable "code" is a real magnet for publications. The challenge is to provide a physical interpretation of the constant with mathematical results matching the number.

Looking at the reciprocal of the Fine Structure constant value of **137.035999…** measured with great precision (one part per 3 billion) it is unfeasible to make an immediate statement where those numbers come from. Even integer portion **137** of the constant very likely does not have any analogy in objects or parameters of 4-dimensional world we know (3 spatial dimensions plus the Time).

But by switching to an assumption that there are extra spatial dimensions in our continuum it is relatively simpler to generate a basis for rationalization of different physical phenomena including the source of the Fine Structure constant's value. Author believes that space itself and specifically number of dimensions are the keys for the deciphering of the Fine Structure constant.

One popular hypothesis [1] tells that there are extra 7 curved at small scale of distances (about Bohr radius) which we cannot sense directly because dimensions are bent with a radius about the size of an atom and resolutions of human vision is not sufficient to observe small details. These 7 dimensions should exist in addition to 3 dimensions we can feel. Total of 10 spatial dimensions is accompanied by the 11-th dimension – the Time.

Using this differentiation of spatial dimensions (**10** dimensions contain **7** dimensions which diverse from the remaining **3** dimensions) author has noticed that a boundary of 10-dimentional hypercube contains $\mathbf{2^{10-7}} \cdot \binom{\mathbf{10}}{\mathbf{7}} = \mathbf{960}$ of 7-dimensional cubes while the decremented value 960-1 = 959 is multiple of **137**.

For short, the origin of **137** can be traced to the 10-dimensional configuration of space and summarized in expression (1) as follows:

$$\mathbf{137} = \frac{\mathbf{959}}{\mathbf{7}} \qquad (1)$$

where

$$\mathbf{959} = \mathbf{960} - \mathbf{1} = \mathbf{2}^{\mathbf{10}-\mathbf{7}} \cdot \binom{\mathbf{10}}{\mathbf{7}} - \mathbf{1} = \mathbf{2}^{\mathbf{10}-\mathbf{7}} \cdot \frac{\mathbf{10}!}{\mathbf{7}!\,(\mathbf{10}-\mathbf{7})!} - \mathbf{1}$$

- $\mathbf{960}$ - is the number of 7-dimensional cubes in 10-dimensional hypercube boundary;
- $\boldsymbol{7}$ - represents the number of small (curved) spatial dimensions;
- 10 - represents the total number of spatial dimensions.

Author also suggests expression (2) for calculation of precise value for the Fine Structure constant reciprocal using the same numbers from 10 dimensional hypercube as parameters:

$$\mathbf{137.035999032} = \frac{\frac{\mathbf{959}}{\mathbf{7}}}{\sqrt{\mathbf{1} - \frac{\boldsymbol{\pi}^{\mathbf{2}}}{\boldsymbol{u} - \mathbf{1}}}} \;, \boldsymbol{where}\ \boldsymbol{u} = \frac{\mathbf{959}}{\mathbf{7}} \cdot \frac{\mathbf{960}}{\mathbf{7}} \qquad (2)$$

The result of the expression (2) resides in the boundaries of 2010 CODATA recommended value for the Fine Structure constant.

Another precise formula (3) contains only integer numbers as parameters

$$\frac{\mathbf{1}}{\mathbf{137.035999040}} = \boldsymbol{\beta} \cdot \frac{\mathbf{7}}{\mathbf{959}} + (\mathbf{1} - \boldsymbol{\beta}) \cdot \frac{\mathbf{7}}{\mathbf{960}} \qquad (3)$$

where

$\boldsymbol{\beta} = \frac{\mathbf{3}}{\mathbf{4}} - \frac{\mathbf{3}}{\mathbf{10 \cdot 137}}$ is the probability for scenario with 959 cells involved;

$\mathbf{1} - \boldsymbol{\beta} = \frac{\mathbf{1}}{\mathbf{4}} + \frac{\mathbf{3}}{\mathbf{10 \cdot 137}}$ is the probability for scenario with 960 cells involved.

Detailed Explanation for the Fine Structure constant origin

The Fine Structure constant is defined as a ratio of the electrostatic energy between two electrons at a distance **d** and energy of photon with the same wavelength **d**.

Considering that we also can inversely tell that the Fine Structure constant is the ratio of the distance **d** between two charged particles and wavelength **D** of photon having the same energy as charged particles at the distance **d**. But photons are generated by charged particles when particles change speed. The question is why in experiments **d** is “seen” as **D** which is **≈137** times longer?

Author’s explanation suggests that the process of this magnification might be similar to cinema’s principle: small image from a film is projected to a big screen and thus expanded. In case of the Fine Structure constant, a charged particle is a projector and screen is hypercube of 10 dimensions representing spatial dimensions in our continuum.

One property of any hypercube is recurrence of its boundary implying that the boundary can be composed of cubes of smaller dimensions.

To illustrate the border of hypercube let’s consider 3-dimensional cube (regular cube). Its boundary is formed by 6 sides of squares, while squares are actually 2-dimensional cubes. Each square in turn has own boundary which consists of 4 line segments or 1-dimensional cubes. Using similar steps but in opposite direction we can compose n-dimensional hyper cube consisting of m-dimensional cubes.

In general, the number of **m**-dimensional hyper cubes on the border of **n**-dimensional hypercube is determined by formula (derived by simple recurrence counting)

$$\boldsymbol{E_{m,n} = 2^{n-m} \cdot \binom{n}{m} = 2^{n-m} \cdot \frac{n!}{m!\,(n-m)!}} \qquad \mathbf{(4)}$$

In the abstract author has promised to explain a way of getting number 137 (associated with integer portion of the Fine Structure constant reciprocal) from properties of 10-dimensional hypercube. The choice of 10 is dictated by total number of spatial dimensions which is hypothetically 10 = 3 + 7. We definitely observe 3 spatial dimensions unless our senses lie to us. Remaining 7 spatial dimensions must be different from 3 “normal” spatial dimensions. Otherwise we would recognize and sense all 10 of them without differentiation. It could be a case these 7 dimensions are too tiny for humans to feel. Nevertheless, diversity of spatial dimensions should be detectable in physical phenomena especially at distances smaller than Bohr radius. And the value of the Fine Structure constant is a solid confirmation for the hypothesis which is limited so far by considering a co-existence of 7 extra dimensions in addition to our good old 3 dimensions.

To elaborate that idea, let staff numbers 10 and 7 in formula (4) to get a count of 7-dimensional hyper cubes on the boundary of 10-cube

$$\boldsymbol{E_{7,10} = 960} \qquad (5)$$

Fig. 1 presents an illustration for boundary of 10-dimensional cube. It consists of 960 7-dimensional cubes.

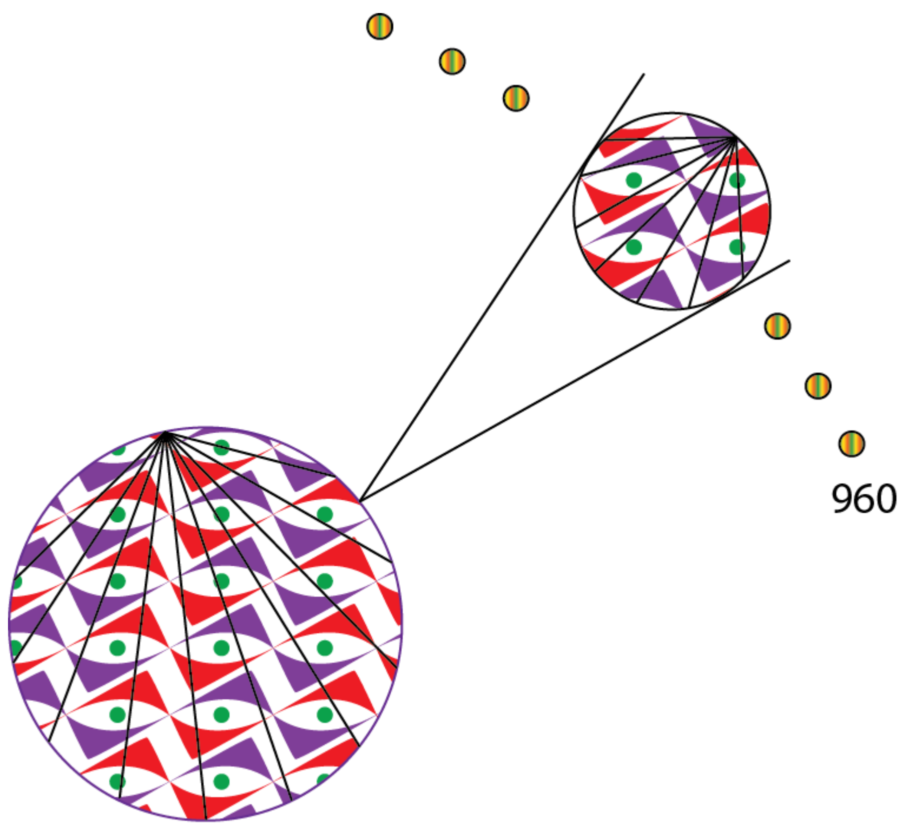

Fig.1 A 10 dimensional cube consists of 960 7-dimensional cubes

Assume that a charged particle happened to be in one of 960 of 7-dimensional cells and this particle emits a photon. Due to its wave nature photon is distributed (projected) on all other cells (960-1=959) with the exception of originating one. Respectively, 959 cells are forming a screen and originating cell is performing a role of a projector.

On the other hand, photon is a corpuscle and prefers to move along straight line and cannot stay. A straight line (edge of the hypercube) is shared by seven adjacent 7-dimensional cubes.

Combining the above considerations into ratio (6) we are deriving an integer portion of the Fine Structure constant.

$$\mathbf{137 = \frac{960 - 1}{7} = \frac{E_{7,10} - 1}{7}} \qquad (6)$$

How to get a precise value for the Fine Structure constant?

Describing method of projection on 959 7-dimensinal cells author has excluded the originating cell (projector) from being part of the screen. In reality, projector can illuminate itself. If photon is distributed on all 960 7-cubes the ratio gives next zoom coefficient of 960/7 = 137.14…

If there are two projection's scenarios (one with zoom ratio 959/7=137 and another with zoom ratio 960/7=137.14…) it should be a probability distribution between them.

The known from experiments value of the Fine Structure constant is positioned in between these two limits 137 and 137.14… but not in the middle. Experimental value corresponds to approximately 75% chance for 959 cells illumination by wave function of photon and 25% chance for all 960 cells illumination.

More precisely,

$$\mathbf{\frac{1}{137.035999040} = \beta \cdot \frac{7}{959} + (1 - \beta) \cdot \frac{7}{960}} \qquad (3)$$

where

$\boldsymbol{\beta = \frac{3}{4} - \frac{3}{10 \cdot 137}}$ is the probability for scenario with 959 cells involved;

$\boldsymbol{1 - \beta = \frac{1}{4} + \frac{3}{10 \cdot 137}}$ is the probability for scenario with 960 cells involved.

Expression (3) produces the result precisely located in the limits for the 2010 CODATA recommended value for the Fine structure constant.

There is a different form of representation for the Fine Structure constant which has been given in the abstract's expression (2) and repeated here. The result (2) is similar (superficially) to a relativistic mass correction for a particle moving at approximately **π/137** of speed of light.

$$\mathbf{137.035999032} = \frac{\frac{959}{7}}{\sqrt{1-\frac{\pi^2}{u-1}}}\ , where\ u = \frac{959}{7}\cdot\frac{960}{7} \qquad (2)$$

Expression (2) has been found by reconstruction of the value of the Fine Structure constant using Taylor series. The result also precisely fits the boundaries for the 2010 CODATA recommended value for the Fine structure constant.

It is significant that formula (2) contains only one hypothetical term which is **7** as the number of extra spatial dimensions. Number **7** produces number **10** as the total number of spatial dimensions (**7** invisible dimensions plus **3** dimensions human can sense).

Number **960** is a count of **7** dimensional cells in **10** dimensional hypercube. Number **959** is decremented version of **960**.

Part 2. Proton-to-electron Mass Ratio and Extra Dimensions

Our experience tells us that mass of an object can take any value. It is possibly true statement for macro world. But if somebody would attempt dividing matter in progressively smaller pieces this process eventually ends up with only few stable particles: predominantly protons and electrons. The rest masses of proton and electron are different but fixed in respect to each other.

The ratio of proton's mass to that of electron or proton-to-electron mass ratio has CODATA recommended value of $\boldsymbol{\mu}$ = 1836.15267245(75). The value is based on results of different experiments and has uncertainty 0.4 parts per billion.

This chapter presents a derivation of proton-to-electron mass ratio $\boldsymbol{\mu}$ through **10**-dimensional geometry projected to **7** dimensions.

The result for the proton-to-electron mass ratio can be summarized in next expression

$$\mu = \frac{(\mathbf{V_{10}}(\mathbf{R}))'''}{\mathbf{R}^7} \cdot \mathbf{k} \tag{7}$$

where

- $\mathbf{V_{10}}$ **(R)** - a volume of **10**-dimensional ball of radius **R**;
  a third derivative of this volume is taken;
- **k** - correctional coefficient equal $\frac{\mathbf{1}}{\mathbf{cos}\left(\mathbf{tanh}(\frac{\mathbf{2\pi}}{\mathbf{2^{10}}})\right)} = \frac{\mathbf{1}}{\mathbf{0.999981175\ldots}}$

Formula (7) produces numerical expression (8)

$$\mu = \frac{\mathbf{6 \cdot \pi^5}}{\mathbf{cos(tanh(\frac{\pi}{512}))}} = \mathbf{1836.15267290\ldots} \tag{8}$$

with the difference between the CODATA value and the result (8) being within boundaries of a standard error for the ratio's measurements.

Derivation of Proton-to-electron Mass Ratio

The rest mass of the particle can be defined as particle's ground level of energy or state with the least possible energiy. It is known from experiments that energy levels take discrete values due to the fact that states (including ground ones) are oscillations. Any oscillation needs a resonance condition in order to exist for some period of time. The better the condition (e.g. less energy loss) the longer oscillation lasts.

But where does magic resonance box with perfect mirrors come from to keep oscillation stable? This is especially interesting question for the ground states of the isolated particles. Why the rest mass does not dissipate in space as the time elapses? What keeps proton and electron unchanged endlessly if they are not affected by other particles or fields?

The answer may come from structure of dimensions. According to the String theory [1] regular **3** dimensions we know and **7** small ones together comprise **10**-dimensional space. The **7** dimensions are so curved that distances along those directions cannot exceed a size of an atom (about Bohr radius). Particles are free to move in **3** regular dimensions but they are effectively contained in a small box (**7**-dimensional hypercube) or small **7**-dimensional ball due to bend of **7** space directions. As result, there is no actual relocation of the particle along these **7** dimensions even in presence of forces. Thus, particle's energy change (being a product of force and a traveled distance) is a zero. The particle cannot gain or loose energy in **7** dimensions and therefore oscillation of the particle in its ground state lasts forever. Particle's rest mass is preserved.

The rest mass of proton is also an oscillation with some geometrical boundaries. The same is applied to electron. As result the ratio of particle's rest masses should also come from this geometry.

For obtaining Proton-to-electron mass ratio numerical value author suggests to analyze a surface area of **10**-dimensional ball as it is seen from **7** dimensions.

Geometrically, first derivative of the volume $\mathbf{V_n(R)}$ of any **n**-dimensional ball is its surface area $\mathbf{S_{n-1}(R)}$ measured in **n-1** dimensions. The relationship of volume and surface area for the ball of radius **R** is described by formula (9)

$$\mathbf{(V_n(R))' = S_{n-1}(R)} \qquad \mathbf{(9)}$$

For example, first derivative of the volume of **3**-dimensional ball (regular ball) has surface area

$$\mathbf{(V_3(R))' = \left(\frac{4}{3} \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}^3\right)' = 4 \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}^2 = S_2(R)} \qquad \mathbf{(10)}$$

This surface area (10) can be expressed in **2**-dimensional units (squares or equivalently **2**-dimensional cubes).

It is known from **n**-dimensional geometry that the volume of **10**-dimensional ball is $\frac{\pi^5}{5!} \cdot R^{10}$.

First derivative of the volume of **10**-dimensional ball equals

$$(\mathbf{V_{10}(R)})' = \left(\frac{\pi^5}{5!} \cdot R^{10}\right)' = \frac{\pi^5}{5!} \cdot 10 \cdot R^9 = \mathbf{S_9(R)} \qquad (11)$$

It is **9**-dimensional surface area of the **10**-dimensional ball.

Second derivative of the volume of **10**-dimensional ball in **8**-dimensions is equal

$$(\mathbf{V_{10}(R)})'' = \left(\frac{\pi^5}{5!} \cdot R^{10}\right)'' = \frac{\pi^5}{5!} \cdot 10 \cdot 9 \cdot R^8 \qquad (12)$$

Finally, third derivative of the volume of **10**-dimensional ball in **7**-dimensions is expressed as

$$(\mathbf{V_{10}(R)})''' = \left(\frac{\pi^5}{5!} \cdot R^{10}\right)''' = \frac{\pi^5}{5!} \cdot 10 \cdot 9 \cdot 8 \cdot R^7 \qquad (13)$$

Considering that **5! = 120** and derivative (13) is calculated in **7**-dimensional units (**7**-dimensional cubes) the equation (13) can be rewritten as follows

$$\frac{(\mathbf{V_{10}(R)})'''}{R^7} = \left(\frac{\pi^5}{5!} \cdot R^{10}\right)''' = 6 \cdot \pi^5 = 1836.1181 \ldots \qquad (14)$$

The value of expression (14) is close to that of the proton-to-electron mass ratio but still needs the dividing coefficient $\cos\left(\tanh\left(\frac{\pi}{512}\right)\right) = 0.999981175 \ldots$ to match experimental results.

This coefficient has been found using Taylor series for the difference between numerical result from expression (14) and CODATA recommended value for Proton-to-electron mass ratio.

The parameter $\frac{\pi}{512} = \frac{2\pi}{2^{10}}$ comes from phase distribution over $\mathbf{2^{10}}$ **= 1024** vertices of **10**-dimensonsal hypercube.

The result of this correction is expressed in equation (8).

Part 3. Law of Gravitation in presence of Extra Dimensions

In this part of the article author suggests an extension for Newton's Law of Gravitation for the distances less than size of an atom where all **10** spatial dimensions (**3** straight + **7** bended) are flat and all contribute to the force of gravity while for larger distances only **3** straight spatial dimensions determine gravitation in accordance with Newton's law.

Author also presents a derivation of Gravitational constant ***G*** as

$$\boldsymbol{G = \frac{\hbar \cdot c}{m^2} \cdot \left(\frac{\alpha}{\mu}\right)^7 \cdot \frac{1}{log_2(2\pi)}} \tag{15}$$

where

- $\hbar$ - reduced Planck constant;
- c - speed of light;
- m - mass of proton;
- α - Fine structure constant;
- μ - proton-to-electron mass ratio.

Formula (1) yields value of ***G*** = **6.67430606(67)·10^{-11} m^3·kg^{-1}·s^{-2}** with relative uncertainty **10^{-7}** which is 3 orders better than uncertainty of CODATA recommended value ***G*** = **6.67384(80)·10^{-11} m^3·kg^{-1}·s^{-2}** for Newtonian constant of Gravitation.

Law of gravitation background and hypothesis

Experimentally confirmed Newton's law of gravitation gives no doubt in its validity on human scale of distances and above

$$\mathbf{F} = -\boldsymbol{G} \cdot \frac{\boldsymbol{M_1} \cdot \boldsymbol{M_2}}{\boldsymbol{r}^2} \qquad (16)$$

where

- **F** - force of gravitation between to masses M_1 and M_2;
- G - gravitational constant $\mathbf{6.67384..\cdot 10^{-11}\, m^3 \cdot kg^{-1} \cdot s^{-2}}$;
- r – distance between masses.

According to formula (16) Gravitational force is inverse proportional to the square of a distance between two masses.

But is this law applicable for all distances? What if for distances smaller than size of an atom the gravitational force becomes inverse proportional to the 9-th power of a distance?

$$\mathbf{F} = -\mathbb{\Gamma} \cdot \frac{\boldsymbol{M_1} \cdot \boldsymbol{M_2}}{\boldsymbol{r}^9} \qquad (17)$$

- where $\mathbb{\Gamma}$ - gravitational constant for distances smaller than size of an atom (a size of an atom can be characterized by Bohr radius $a_0 \approx 5.29... \cdot 10^{-11}$ m).

9-th power of a distance in equation (17) corresponds to **10** spatial dimensions similar to 2-d power of a distance in equation (16) for **3** spatial dimensions.

In other words, as a distance between two objects (particles) gets smaller, the curved seven extra dimensions become flat on the scale of shorter distances and contribute more to the force of gravitation. All **10** spatial dimensions work and gravitation becomes stronger. Is it Strong force itself?

To make an estimation for gravitational constant $\mathbb{\Gamma}$ for subatomic distances we can use a point of transition from $\sim 1/r^2$ law of gravitation for distances bigger than Bohr radius a_0 to the proposed law of gravitation $\sim 1/r^9$ for distances less than Bohr radius a_0. At a distance r equal Bohr radius a_0 the forces depicted in formula (16) and (17) should be approximately equal to each other.

By equalizing expressions (16) and (17) and substituting $\mathbf{r}$ for $\mathbf{a}_0$ result for $\mathbb{\Gamma}$ is determined by following formula (18).

$$\mathbb{\Gamma} \sim \boldsymbol{G} \cdot \boldsymbol{a}_0^7 \qquad (\mathbf{18})$$

where

- $\boldsymbol{G}$ - gravitational constant **6.67384..·10^{-11} m^3·kg^{-1}·s^{-2}**;
- $a_0 \approx 5.29... \cdot 10^{-11}$ m – Bohr radius.

The formula (18) yields value about $0.8... \cdot 10^{-82}$ **m^{10}·kg^{-1}·s^{-2}** for gravitational constant $\mathbb{\Gamma}$ at subatomic distances.

Interpolation plot Fig.2 shows that gravitational constant for subatomic distances should be bigger than value $\mathbb{\Gamma}$ calculated from equation (18).

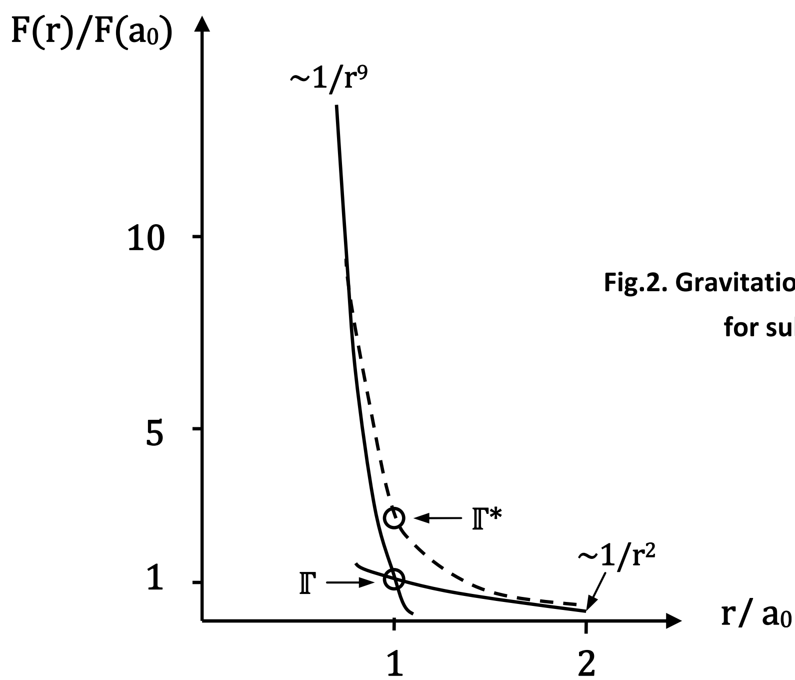

Fig.2. Gravitational force F vs. distance r for subatomic range.

Transition (shown as dashed line) from $\sim 1/r^2$ to $\sim 1/r^9$ law of gravitation happens in the range half of Bohr radius. The magnitude of gravitational constant $\mathbb{\Gamma}^*$ is closer to $2.0564... \cdot 10^{-82}$ **m^{10}·kg^{-1}·s^{-2}** and can be expressed as

$$\mathbb{\Gamma}^* = \boldsymbol{log_2(2\pi)} \cdot \boldsymbol{G} \cdot \boldsymbol{a}_0^7 \qquad (\mathbf{19})$$

Coefficient $\log_2(2\pi)$ in equation (19) represents the entropy of the system with probability to occupy one of the states equal $1/(2\pi)$.

The neutron-to-proton mass ratio is a confirmation that the same coefficient $\log_2(2\pi)$ is “used” by nature elsewhere besides Gravitational constant

$$\frac{m_n}{m} = \frac{1}{\sqrt{1 - \frac{\alpha}{log_2(2\pi)}}} \approx 1 + \frac{\alpha}{2 \cdot log_2(2\pi)} + \ \dots \qquad [20]$$

where

- m_n - mass of neutron;
- m - mass of proton;
- α - Fine structure constant.

Determination of Gravitational constant $\mathbb{\Gamma}^*$ for subatomic scale and Newtonian Gravitational constant G

Humans can sense **4** dimensions directly (time and **3** spatial dimensions). The curvature of these **3** spatial dimensions corresponds to the size of Universe $8.80\ldots\cdot 10^{26}$ m. These dimensions are really flat (straight) for us.

According to the String theory [1] there are extra **7** dimensions which are curved with characteristic radius called Bohr radius $a_0 \approx 5.29\ldots\cdot 10^{-11}$ m. It is typical size of an atom. We cannot sense these seven dimensions directly. Yet it is possible to imagine them and prove their existence via measured physical constants.

If observer is much bigger than Bohr radius all extensions of the object in **7** curved dimensions will be seen as a ball no bigger than an atom. Effectively, object’s shape in **7** curved dimensions is not visible to humans due to a big difference in sizes of observer and curvature of these **7** dimensions.

But we can imagine an observer which gets smaller and smaller. The same bended dimension looks progressively flattened to observer as illustrated in Fig.3.

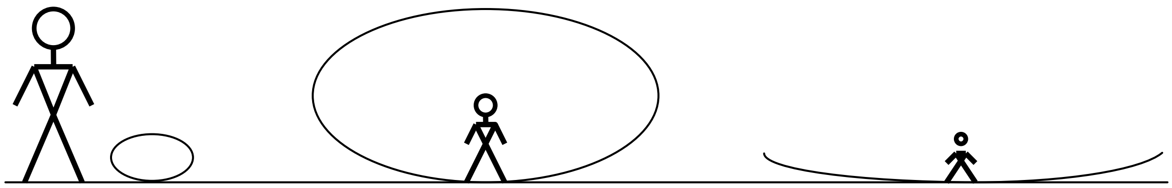

Fig.3. Flattening of the same dimension (represented by oval) as observer gets smaller.

For distances considerably smaller than Bohr radius **7** curved dimensions become essentially straight and observer would notice that all **10** spatial dimensions are equivalent on this scale of distances. For this reason the force of Gravitation hypothetically changes its dependency on a distance from inverse proportional to the distance in power of 2 (at distances more than Bohr radius) to inverse proportional to the distance in power of 9 (for shorter distances).

Equations (16) and (17) comprise two branches of the same Law of Gravitation for two ranges of the distance. Gravitational constant $\boldsymbol{G}$ in equation (16) has been measured directly on scale of meters and above. Gravitational constant $\mathbb{\Gamma}^*$ from equation (17) can be calculated using following sequence of expressions.

A quant of force $\mathbf{F_q}$ for a proton with mass $\boldsymbol{m}$ is a ratio of the particle's energy $\boldsymbol{m \cdot c^2}$ to the reduced de Broglie wavelength $\boldsymbol{\lambda = \hbar/(m \cdot c)}$

$$\mathbf{F_q} = \frac{\boldsymbol{m \cdot c^2}}{\boldsymbol{\lambda}} = \frac{\boldsymbol{m^2 \cdot c^3}}{\boldsymbol{\hbar}} \qquad (21)$$

Gravitational force from equation (17) for the same distance as de Broglie wavelength $\boldsymbol{\lambda}$ equals

$$|\mathbf{F}| = \mathbb{\Gamma}^* \cdot \frac{\boldsymbol{m^2}}{\boldsymbol{\lambda^9}} = \mathbb{\Gamma}^* \cdot \frac{\boldsymbol{m^{11} \cdot c^9}}{\boldsymbol{\hbar^9}} \qquad (22)$$

Now, equalizing quant of force and force of Gravitation $\mathbf{F_q} = |\mathbf{F}|$ we obtain expression for subatomic Gravitational constant $\mathbb{\Gamma}^*$.

$$\mathbb{\Gamma}^* = \frac{\boldsymbol{\hbar^8}}{\boldsymbol{m^9 \cdot c^6}} = \mathbf{2.05639 \ldots \cdot 10^{-82}} \; \boldsymbol{m^{10} kg^{-1} s^{-2}} \qquad (23)$$

Using equation (19) the Newtonian Gravitational constant $\boldsymbol{G}$ can be calculated as well

$$\boldsymbol{G} = \frac{\mathbb{\Gamma}^*}{\boldsymbol{log_2(2\pi)\cdot a_0^7}} = \frac{\hbar^8}{\boldsymbol{m^9\cdot c^6}} \cdot \frac{\boldsymbol{m_e^7\cdot c^7\cdot \alpha^7}}{\hbar^7} \cdot \frac{\mathbf{1}}{\boldsymbol{log_2(2\pi)}} \tag{24}$$

where

- $\hbar$ - reduced Planck constant;
- $\boldsymbol{m}$ - mass of proton;
- $\boldsymbol{a_0 = \hbar/(m_e\cdot c\cdot \alpha)}$ - Bohr radius;
- $\boldsymbol{m_e}$ - mass of electron;
- $\boldsymbol{c}$ - speed of light;
- α - Fine structure constant;
-

After simplification of equation (24) the Newtonian Gravitational constant $\boldsymbol{G}$ is derived as

$$\boldsymbol{G} = \frac{\hbar\cdot\boldsymbol{c}}{\boldsymbol{m^2}}\cdot\left(\frac{\boldsymbol{\alpha}}{\boldsymbol{\mu}}\right)^{\mathbf{7}}\cdot\frac{\mathbf{1}}{\boldsymbol{log_2(2\pi)}} \tag{25}$$

where

- $\hbar$ - reduced Planck constant;
- $\boldsymbol{c}$ - speed of light;
- $\boldsymbol{m}$ - mass of proton;
- α - Fine structure constant;
- $\boldsymbol{\mu}$ - proton-to-electron mass ratio $\boldsymbol{m/m_e}$.

Power of **7** for dimensionless constants α and $\boldsymbol{\mu}$ in equation (25) may reflect the number of curved dimensions in our space-time continuum.

Knowing now both Gravitational constants $\boldsymbol{G}$ (25) and $\mathbb{\Gamma}^*$ (23) the general Law of Gravitation can be summarized (equations (16)+(17)) for all distances $\boldsymbol{r}$ between masses

$$\mathbf{F} = -\,\mathbb{\Gamma}^*\cdot\frac{\boldsymbol{M_1\cdot M_2}}{\boldsymbol{r^9}} - \boldsymbol{G}\cdot\frac{\boldsymbol{M_1\cdot M_2}}{\boldsymbol{r^2}} \tag{26}$$

$$\mathbf{F} = -\,G \cdot \frac{M_1 \cdot M_2}{r^2} \left(log_2(2\pi) \cdot \left(\frac{a_0}{r}\right)^7 + 1 \right) \qquad (27)$$

Presentation (27) for the Law of Gravitation is self-explainable: for distances r bigger than Bohr radius a_0 the Law is just Newtonian Law of Gravitation.

$$\mathbf{F} = -\,\frac{\hbar \cdot c}{m^2} \cdot \left(\frac{\alpha}{\mu}\right)^7 \cdot \frac{1}{log_2(2\pi)} \cdot \frac{M_1 \cdot M_2}{r^2} \cdot \left(log_2(2\pi) \cdot \left(\frac{a_0}{r}\right)^7 + 1 \right) \qquad (28)$$

Equivalent equation (28) for the Law of Gravitation uses the representation (25) for Gravitational constant G which is expressed via other physical constants.

Is proton a black hole?

Lets calculate energy required for a particle of mass M to escape gravity of proton with mass m using $\sim 1/\mathrm{r}^9$ law of Gravitation.

$$\int_{\mathrm{R_s}}^{\infty} \mathbf{F} \cdot \mathbf{dr} = -\int_{R_s}^{\infty} \mathbb{\Gamma}^* \cdot \frac{M \cdot m}{r^9} \cdot dr \;=\; \mathbb{\Gamma}^* \cdot \frac{M \cdot m}{8 \cdot R_s^8} \qquad (29)$$

- where $\mathrm{R_s}$ is initial distance between particles .

Equalizing energy from equation (29) with particle's energy $\mathrm{M{\cdot}c^2}$ the distance $\mathrm{R_s}$ can be expressed as follows.

$$R_s = \sqrt[8]{\mathbb{\Gamma}^* \cdot \frac{m}{8 \cdot c^2}} = \frac{\hbar}{m \cdot c} \cdot \frac{1}{\sqrt[8]{8}} = \lambda \cdot 0.771 \ldots \qquad (30)$$

- where λ - reduced de Broglie wavelength of proton.

$\mathrm{R_s}$ is Schwarzschild radius for proton. Thus, proton can be a self sustaining black hole due to self gravity.

What is the Gravity's propagation speed?

Due to curvature of **7** dimensions the distance between objects for the force of Gravity cannot exceed the Bohr radius. Thus the time of interaction for gravity does not exceed the threshold (31) no matter how far apart objects in **3** flat dimensions.

$$t_G = \frac{a_0}{c} = \frac{\hbar}{m_e \cdot c^2 \cdot \alpha} = 1.765..10^{-19}\ s \qquad (31)$$

Thus, for distances bigger than Bohr radius we can talk only about gravity's propagation time (not exceeding time t_G from expression (31)).

For distances shorter than Bohr radius the speed of Gravity equals speed of light.

Expression for Gravitational coupling constant

Gravitational coupling constant is defined for pairs of elementary particles with masses m_1 and m_2 as follows

$$\alpha_G = \frac{G \cdot m_1 \cdot m_2}{\hbar \cdot c} \qquad (32)$$

It is dimensionless ratio and can be applied to a couple of protons, pair of proton and electron, electron-electron pair and so on.

After substitution G with expression (25) the gravitational coupling constant is expressed as follows.

$$\alpha_{G\ proton-proton} = \frac{\alpha^7}{\mu^7} \cdot \frac{1}{log_2(2\pi)} \qquad (33)$$

$$\alpha_{G\ proton-electron} = \frac{\alpha^7}{\mu^8} \cdot \frac{1}{log_2(2\pi)} \qquad (34)$$

$$\alpha_{G\ electron-electron} = \frac{\alpha^7}{\mu^9} \cdot \frac{1}{log_2(2\pi)} \qquad (35)$$

where

- α - Fine structure constant;
- μ - proton-to-electron mass ratio.

Part 4. Planck Units in Extra Dimensions

Fundamental Planck units (length, mass, time, charge and temperature) are derived from five physical constants (Gravitational constant, Planck constant, Speed of light, Coulomb constant, Boltzmann constant) using dimensional analysis (referring to measuring units).

Despite their popularity among physicists these units in general have no practical use. They are notorious for wild swing of orders of magnitude in respect to observed physical phenomena.

For example, Planck length **$1.616199...\cdot 10^{-35}$ m** is 20 orders smaller than proton charge radius while Planck mass is **$2.17651...\cdot 10^{-8}$ kg** is 19 orders bigger than proton mass **$1.672621777...\cdot 10^{-27}$ kg.**

Planck Temperature **$1.417...\cdot 10^{32}$ K** is so high that Big Bang temperature **10^{13} K** is a very cold event.

This chapter of the article describes how Planck units can be normalized by deriving them in consideration of **10** spatial dimensions in our space-time continuum.

In previous chapter of the article it was shown that an extension of Gravitational law (expression (26)) for distances shorter than an atom may be needed. As result, the Gravitational constant (23) has been suggested. The constant generates new expressions for the Planck units.

Dimensional analysis for normalized Planck units in 10 spatial dimensions

New normalized Planck units for 10 spatial dimensions can be obtained from next physical constants using well known dimensional analysis

$\boldsymbol{c}$ - speed of light 2.99792458 $\cdot 10^{8}$ **m** $\cdot$**s**$^{-1}$

$\mathbb{\Gamma}^{*}$ **-** subatomic gravitational constant from equation (23) 2.05639…$\cdot 10^{-82}$ **m**$^{10}\cdot$**kg**$^{-1}\cdot$**s**$^{-2}$ (instead of Newton's gravitational constant $\boldsymbol{G}$);

$\hbar$ - reduced Planck constant 1.054571726…$\cdot 10^{-34}$ **J** $\cdot$**s**

$\boldsymbol{\varepsilon_0}$ - permittivity of free space 8.854187817620…$\cdot 10^{-12}$ **C** $\cdot$**N**$^{-1}$ $\cdot$**m**$^{-2}$

$\boldsymbol{k_B}$ - Boltzmann constant 1.3806488…$\cdot 10^{-23}$ **J** $\cdot$**K**$^{-1}$

Results of dimensional analysis are next.

Fundamental Planck units normalized in 10 spatial dimensions

Normalized Planck length

$$\boldsymbol{l_{\circlearrowright}} = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}^{*}}{\boldsymbol{c^3}}} = \mathbf{2.103089} \ldots \cdot \mathbf{10^{-16}}\, \boldsymbol{m} \qquad \mathbf{(36)}$$

It is reduced Compton wavelength for proton.

Normalized Planck mass

$$\boldsymbol{m_{\circlearrowright}} = \sqrt[9]{\frac{\hbar^{\mathbf{8}}}{\mathbb{\Gamma}^{*} \cdot \boldsymbol{c^6}}} = \mathbf{1.864042} \ldots \cdot \mathbf{10^{-27}}\, \boldsymbol{kg} \qquad \mathbf{(37)}$$

It is mass of proton.

Normalized Planck time

$$\boldsymbol{t_{\circlearrowright} = \sqrt[9]{\frac{\hbar \cdot \mathbb{\Gamma}^*}{c^{12}}} = 7.015150 \ldots \cdot 10^{-25}\, s} \qquad (38)$$

Time required for light to travel a distance of reduced Compton wavelength for proton.

Normalized Planck temperature

$$\boldsymbol{T_{\circlearrowright} = \frac{1}{k_B} \cdot \sqrt[9]{\frac{\hbar^8 \cdot c^{12}}{\mathbb{\Gamma}^*}} = 1.0888196 \ldots \cdot 10^{13}\, K} \qquad (39)$$

It is an order of Big Bang temperature during first second.

Other normalized Planck units in 10 spatial dimensions

Normalized Planck momentum

$$\boldsymbol{m_{\circlearrowright} \cdot c = \sqrt[9]{\frac{\hbar^8 \cdot c^3}{\mathbb{\Gamma}^*}} = 5.014394 \ldots \cdot 10^{-19}\, kg \cdot m \cdot s^{-1}} \qquad (40)$$

It is a momentum for proton moving at speed close to that of light.

Normalized Planck energy

$$\boldsymbol{E_{\circlearrowright} = m_{\circlearrowright} \cdot c^2 = \sqrt[9]{\frac{\hbar^8 \cdot c^{12}}{\mathbb{\Gamma}^*}} = 1.503277 \ldots \cdot 10^{-10}\, J} \qquad (41)$$

It is energy of proton at rest.

Normalized Planck force

$$\boldsymbol{F}_{\circlearrowright} = \frac{\boldsymbol{E}_{\circlearrowright}}{\boldsymbol{l}_{\circlearrowright}} = \sqrt[9]{\frac{\hbar^{7} \cdot \boldsymbol{c}^{15}}{\mathbb{\Gamma}^{*2}}} = \mathbf{7.147950} \ldots \cdot \mathbf{10^{5}}\ \boldsymbol{N} \qquad (42)$$

It is energy of proton at rest divided by normalized Planck length.

Normalized Planck power

$$\boldsymbol{P}_{\circlearrowright} = \frac{\boldsymbol{E}_{\circlearrowright}}{\boldsymbol{t}_{\circlearrowright}} = \sqrt[9]{\frac{\hbar^{7} \cdot \boldsymbol{c}^{24}}{\mathbb{\Gamma}^{*2}}} = \mathbf{2.142901} \ldots \cdot \mathbf{10^{14}}\ \boldsymbol{W} \qquad (43)$$

It is energy of proton at rest divided by normalized Planck time.

Normalized Planck angular frequency

$$\boldsymbol{\omega}_{\circlearrowright} = \frac{\mathbf{1}}{\boldsymbol{t}_{\circlearrowright}} = \sqrt[9]{\frac{\boldsymbol{c}^{12}}{\hbar \cdot \mathbb{\Gamma}^{*}}} = \mathbf{1.425486} \ldots \cdot \mathbf{10^{24}}\ \boldsymbol{s}^{-1} \qquad (44)$$

It is a reciprocal of normalized Planck time.

Conclusion

Four preceding parts of the article allow outlining several intriguing conclusions.

Taking into consideration extra 7 spatial dimensions in addition to 3 observable dimensions it is possible to suggest physical interpretations and derive numerical values for the next physical constants:

Fine Structure constant	-	expressions	(2), (3);
Proton-to-electron mass ratio	-	expression	(8);
Gravitational constant	-	expression	(15);
Gravitational coupling constants	-	expressions	(33), (34), (35).

The gravitational law for subatomic distances (where all 10 spatial dimensions are flat) can be described by equation (27) resulting in the conclusion that "strong" force is just gravitation, because for distances shorter than reduced Compton wavelength for proton the gravitation exceeds electrostatic force.

Using suggested variant of gravitational law (27) it is feasible to show that proton due to self gravity can form a black hole with radius about reduced Compton wavelength (expression (30)).

Applying new interpretation of Gravitational constant for subatomic distances (expression (23)), Plank units get normalization (expressions (36)… (44)) to the scale of proton.

References

1. Duff, Michael J., M-Theory (the Theory Formerly Known as Strings), International Journal of Modern Physics A, 11 (1996) 5623–5642, online at Cornell University's arXiv ePrint server