Law of Gravitation in 11 Dimensions

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October 16, 2011

Abstract

The article challenges Newton's Law of Gravitation for distances less than size of an atom where all 11 dimensions are flat and contribute to the force of gravity.

Author also presents a derivation of Gravitational constant $\boldsymbol{G}$ as

$$\boldsymbol{G = \frac{\hbar \cdot c}{m^2} \cdot \left(\frac{\alpha}{\mu}\right)^7 \cdot \frac{1}{log_2(2\pi)}} \qquad [\mathbf{1}]$$

where

- $\boldsymbol{\hbar}$ - reduced Planck constant;
- $\boldsymbol{c}$ - speed of light;
- $\boldsymbol{m}$ - mass of proton;
- α - Fine structure constant;
- $\boldsymbol{\mu}$ - proton-to-electron mass ratio.

Formula [1] yields value of $\boldsymbol{G}$ = **6.67430606(67)·10^{-11} m^3·kg^{-1}·s^{-2}** with relative uncertainty **10^{-7}** which 3 orders better than uncertainty of CODATA recommended value $\boldsymbol{G}$ = **6.67384(80)·10^{-11} m^3·kg^{-1}·s^{-2}** for Newtonian constant of Gravitation.

Newton's law of gravitation in 11 dimensions

Experimentally confirmed Newton's law of gravitation gives no doubt in its validity on human scale distances and above

$$\mathbf{F} = -\,\boldsymbol{G} \cdot \frac{\boldsymbol{M_1} \cdot \boldsymbol{M_2}}{\boldsymbol{r^2}} \qquad [2]$$

where

- **F** - force of gravitation between to masses $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$;
- $\boldsymbol{G}$ - gravitational constant $\mathbf{6.67384..\cdot 10^{-11}\ m^3\cdot kg^{-1}\cdot s^{-2}}$;
- $\boldsymbol{r}$ – distance between masses.

According to formula [2] Gravitational force is inverse proportional to the square of a distance between two masses.

But is this law applicable for all distances? What if for distances smaller than size of an atom the gravitational force becomes inverse proportional to the 9-th power of a distance?

$$\mathbf{F} = -\,\mathbb{\Gamma} \cdot \frac{\boldsymbol{M_1} \cdot \boldsymbol{M_2}}{\boldsymbol{r^9}} \qquad [3]$$

- where $\mathbb{\Gamma}$ - gravitational constant for distances smaller than size of an atom (a size of an atom can be characterized by Bohr radius $a_0 \approx 5.29...\cdot 10^{-11}\,\mathrm{m}$).

9-th power of a distance in equation [3] corresponds to 11 dimensions similar to 2-d power of a distance in equation [2] for 4 dimensions (3 spatial dimensions and the time).

In other words, as a distance gets smaller, curved seven extra dimensions become flat on this scale and contribute more to the force of gravitation. All 11 dimensions work and gravitation becomes stronger. Is it Strong force itself?

To make estimation for gravitational constant $\mathbb{\Gamma}$ for subatomic distances let assume that forces in equation [2] and equation [3] are equal to each other at a distance of Bohr radius a_0. This assumption results in expression [4].

$$\mathbb{\Gamma} \sim \boldsymbol{G} \cdot \boldsymbol{a_0^7} \qquad [4]$$

It yields value about $0.8 \cdot 10^{-82}$ $\mathbf{m^{10}\cdot kg^{-1}\cdot s^{-2}}$.

Interpolation plot Fig.1 shows that gravitational constant for subatomic distances should be bigger than value $\mathbb{\Gamma}$ calculated from equation [4].

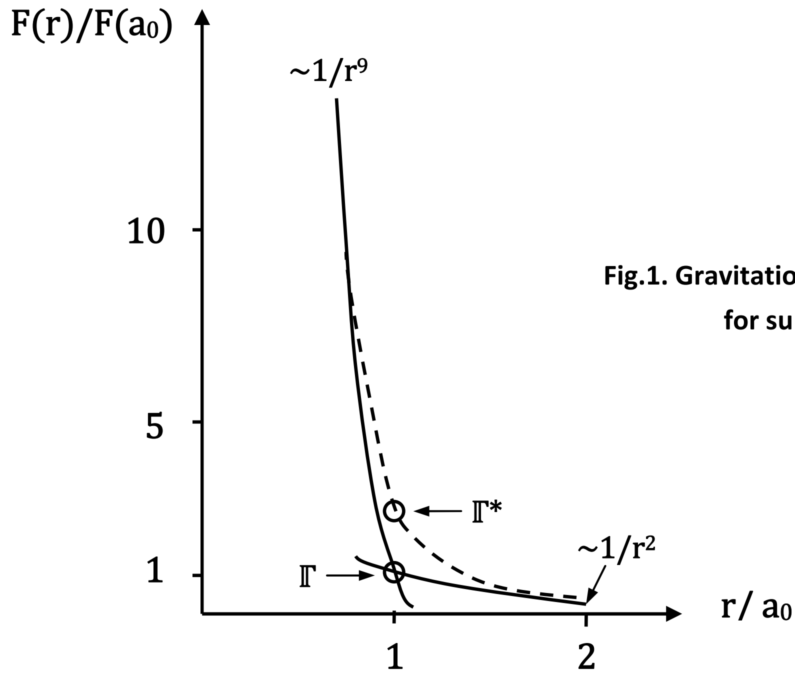

Fig.1. Gravitational force F vs. distance r for subatomic range.

Transition (shown as dashed line) from $\sim 1/r^2$ to $\sim 1/r^9$ law of gravitation happens in the range half of Bohr radius. The magnitude of gravitational constant $\mathbb{\Gamma}^*$ will be closer to $2.0564\ldots \cdot 10^{-82}$ $\mathbf{m^{10} \cdot kg^{-1} \cdot s^{-2}}$ and can be expressed as

$$\mathbb{\Gamma}^* = \boldsymbol{log_2(2\pi) \cdot G \cdot a_0^7} \qquad [5]$$

Coefficient $\log_2(2\pi)$ in equation [5] represent the entropy of the system with probability to occupy one of the states equal $1/(2\pi)$.

The neutron-to-proton mass ratio is a confirmation that the same coefficient $\log_2(2\pi)$ is "used" by nature elsewhere besides Gravitational constant

$$\boldsymbol{\frac{m_n}{m_p} = \frac{1}{\sqrt{1 - \frac{\alpha}{log_2(2\pi)}}} \approx 1 + \frac{\alpha}{2 \cdot log_2(2\pi)} + \ \ldots} \qquad [6]$$

where

- m_n - mass of neutron;

- $\boldsymbol{m_p}$ - mass of proton;
- α - Fine structure constant.

Determination of Gravitational constant $\mathbb{\Gamma}^*$ for subatomic scale

Humans can sense 4 dimensions (time and 3 spatial dimensions). The curvature of these 3 spatial dimensions corresponds to the size of Universe $8.80...\cdot 10^{26}$ m. These dimensions are really flat (straight) for us.

According to the M-theory (String theory) (ref.1) there are extra 7 dimensions which are curved with characteristic radius called Bohr radius $a_0 \approx 5.29...\cdot 10^{-11}$ m. It is typical size of an atom. We can not sense these seven dimensions directly. Yet it is possible to imagine them and prove their existence via physical constants measured on scale of meters and above.

If observer is much bigger than Bohr radius all extensions of the object in 7 curved dimensions will be seen as a ball no bigger than an atom. Effectively, object's shape in 7 curved dimensions is not visible to humans due to big difference in sizes of observer and curvature of these 7 dimensions.

But we can imagine an observer which gets smaller and smaller. The same curved dimension looks progressively flattened to observer as illustrated in Fig.2.

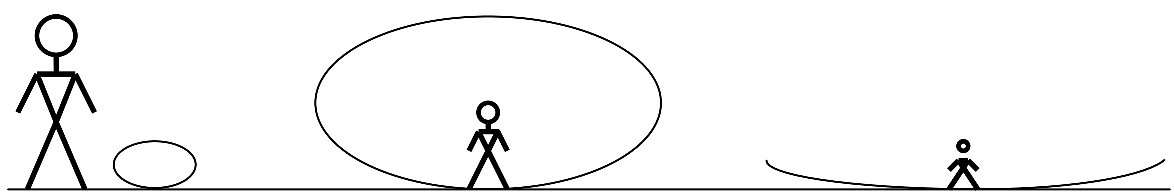

Fig.2. Flattening of the same dimension (represented by oval) as observer gets smaller.

For distances considerably smaller than Bohr radius all 10 spatial dimensions are essentially flat and Force of Gravitation changes its Law from inverse proportional to the distance in power of 2 (at distances more than Bohr radius) to inverse proportional to the distance in power of 9 (for smaller distances). It happens because all 10 spatial dimensions are equivalent on smaller scale of distances.

Equations [2] and [3] comprise two branches of the same Law of Gravitation for two ranges of the distance. Gravitational constant $\boldsymbol{G}$ in equation [2] has been measured directly on scale of meters and above. Gravitational constant $\mathbb{\Gamma}^*$ from equation [3] can be calculated as follows.

For a proton with mass m a quant of force F_q is a ratio of the particle's energy $m \cdot c^2$ to the reduced de Broglie wavelength $\lambda = \hbar/(m \cdot c)$

$$\mathbf{F_q} = \frac{m \cdot c^2}{\lambda} = \frac{m^2 \cdot c^3}{\hbar} \qquad [7]$$

Gravitational force from equation [3] for the same distance as de Broglie wavelength λ equals

$$|\mathbf{F}| = \Gamma^* \cdot \frac{m^2}{\lambda^9} = \Gamma^* \cdot \frac{m^{11} \cdot c^9}{\hbar^9} \qquad [8]$$

Now, equalizing quant of force and force of Gravitation $F_q = |F|$ we can obtain expression for subatomic Gravitational constant Γ^*.

$$\Gamma^* = \frac{\hbar^8}{m^9 \cdot c^6} \qquad [9]$$

Using equation [5] the Newtonian Gravitational constant G can be calculated as well

$$G = \frac{\Gamma^*}{log_2(2\pi) \cdot a_0^7} = \frac{\hbar^8}{m^9 \cdot c^6} \cdot \frac{m_e^7 \cdot c^7 \cdot \alpha^7}{\hbar^7} \cdot \frac{1}{log_2(2\pi)} \qquad [10]$$

where

- $\hbar$ - reduced Planck constant;
- m - mass of proton;
- $a_0 = \hbar/(m_e \cdot c \cdot \alpha)$ - Bohr radius;
- m_e - mass of electron;
- c - speed of light;
- α - Fine structure constant;
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After simplification of equation [10] the Newtonian Gravitational constant G is derived as

$$G = \frac{\hbar \cdot c}{m^2} \cdot \left(\frac{\alpha}{\mu}\right)^7 \cdot \frac{1}{log_2(2\pi)} \qquad [11]$$

where

- $\hbar$ - reduced Planck constant;
- c - speed of light;
- m - mass of proton;
- α - Fine structure constant;
- μ - proton-to-electron mass ratio m/m_e.

Power of 7 for dimensionless constants α and μ in equation [11] actually reflects the number of curved dimensions in our space-time continuum.

Knowing now both Gravitational constants G [11] and $\mathbb{\Gamma}^*$ [9] the general Law of Gravitation can be summarized (equations [2]+[3]) for all distances r between masses

$$\mathbf{F} = -\,\mathbb{\Gamma}^* \cdot \frac{M_1 \cdot M_2}{r^9} - G \cdot \frac{M_1 \cdot M_2}{r^2} \qquad [12]$$

$$\mathbf{F} = -\,G \cdot \frac{M_1 \cdot M_2}{r^2} \left(log_2(2\pi) \cdot \left(\frac{a_0}{r}\right)^7 + 1 \right) \qquad [13]$$

Presentation [13] for the Law of Gravitation is self-explainable: for distances r bigger than Bohr radius a_0 the Law is just Newtonian Law of Gravitation.

$$\mathbf{F} = -\,\frac{\hbar \cdot c}{m^2} \cdot \left(\frac{\alpha}{\mu}\right)^7 \cdot \frac{1}{log_2(2\pi)} \cdot \frac{M_1 \cdot M_2}{r^2} \cdot \left(log_2(2\pi) \cdot \left(\frac{a_0}{r}\right)^7 + 1 \right) \qquad [14]$$

Equivalent equation [14] for the Law of Gravitation uses the representation [11] for Gravitational constant G which is expressed via other physical constants.

Is proton a black hole?

Lets calculate energy required for a particle of mass M to escape gravity of proton with mass m using $\sim 1/\mathrm{r}^9$ law of Gravitation.

$$\int_{\mathbf{R_s}}^{\infty} \mathbf{F} \cdot \mathbf{dr} = - \int_{R_s}^{\infty} \Gamma^* \cdot \frac{M \cdot m}{r^9} \cdot dr = \Gamma^* \cdot \frac{M \cdot m}{8 \cdot R_s^8} \qquad [15]$$

- where $\mathrm{R_s}$ is initial distance between particles .

Equalizing energy from equation [15] with particle's energy $\mathrm{M \cdot c^2}$ the distance $\mathrm{R_s}$ can be expressed as follows.

$$R_s = \sqrt[8]{\Gamma^* \cdot \frac{m}{8 \cdot c^2}} = \frac{\hbar}{m \cdot c} \cdot \frac{1}{\sqrt[8]{8}} = \lambda \cdot 0.771 \ldots \qquad [16]$$

- where λ - reduced de Broglie wavelength of proton.

$\mathrm{R_s}$ is Schwarzschild radius for proton. Thus, proton can be a self sustaining black hole due to self gravity.

What is the Gravity's propagation speed?

Due to curvature of 7 dimensions the distance between objects for the force of Gravity can not exceed the Bohr radius. Thus the time of interaction for gravity does not exceed the threshold [17] no matter how far apart objects in 3 flat dimensions

$$t_G = \frac{a_0}{c} = \frac{\hbar}{m_e \cdot c^2 \cdot \alpha} = 1.765..10^{-19}\, s \qquad [17]$$

Thus, for distances bigger than Bohr radius one can talk only about gravity's propagation time (not exceeding time t_G from expression [17]).

For distances shorter than Bohr radius the speed of Gravity equals speed of light.

References

1. Duff, Michael J., M-Theory (the Theory Formerly Known as Strings), International Journal of Modern Physics A, 11 (1996) 5623–5642, online at Cornell University's arXiv ePrint server